Features

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The Input Space $\mathcal{X}$

- Our general learning theory setup: no assumptions about $\mathcal{X}$
- But $\mathcal{X} = \mathbf{R}^d$ for the specific methods we've developed:
    - Ridge regression
    - Lasso regression
    - Linear SVM

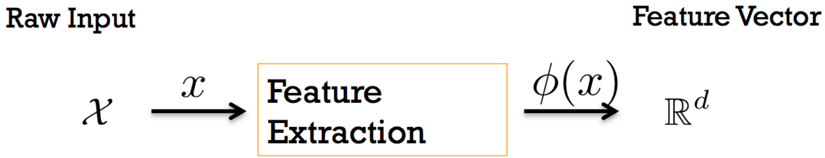
The Input Space $\mathcal{X}$

- Often want to use inputs not natively in $\mathbf{R}^d$:
  - Text documents
  - Image files
  - Sound recordings
  - DNA sequences
- But everything in a computer is a sequence of numbers?
  - The ith entry of each sequence should have the same "meaning"
  - All the sequences should have the same length

Feature Extraction

Definition

Mapping an input from $\mathcal{X}$ to a vector in $\mathbf{R}^d$ is called **feature extraction** or **featurization**.



Example: Detecting Email Addresses

- Task: Predict whether a string is an email address
- Could use domain knowledge and write down:

abc@gmail.com $\xrightarrow[\text{arbitrary!}]{\textbf{feature extractor}}$

length>10	: 1
fracOfAlpha	: 0.85
contains_@	: 1
endsWith_com	: 1
endsWith_org	: 0

- But this was ad-hoc, and maybe we missed something.
- Could be more systematic?

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Feature Templates

Definition (informal)

A **feature template** is a group of features all computed in a similar way.

- Input: *abc@gmail.com*

Feature Templates

- Length greater than ___
- Last three characters equal ___
- Contains character ___

Based on Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Feature Template: Last Three Characters Equal ___

- Don't think about which 3-letter suffixes are meaningful...
- Just **include them all.**

abc@gmail.com

endsWith_aaa : 0
endsWith_aab : 0
endsWith_aac : 0
...
endsWith_com : 1
...
endsWith_zzz : 0

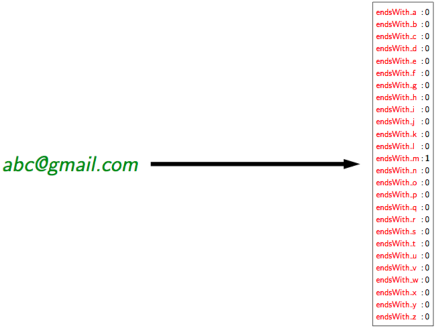
- With regularization, our methods will not be overwhelmed.

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Feature Template: One-Hot Encoding

Definition

A **one-hot encoding** is a set of features (e.g. a feature template) that always has **exactly one** non-zero value.



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Feature Vector Representations

fracOfAlpha : 0.85
contains_a : 0
...
contains_@ : 1
...

Array representation (good for dense features):

```
[0.85, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]
```

Map representation (good for sparse features):

```
{"fracOfAlpha": 0.85, "contains_@": 1}
```

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Feature Vector Representations

- Arrays
  - assumed fixed ordering of the features
  - appropriate when significant number of nonzero elements ("**dense feature vectors**")
  - very efficient in space and speed (and you can take advantage of GPUs)
- Map (a "dict" in Python)
  - best for **sparse feature vectors** (i.e. few nonzero features)
  - features not in the map have default value of zero
  - Python code for "ends with last 3 characters":

    ```
    {"endsWith "+x[-3:]: 1}.
    ```

  - Has overhead compared to arrays, so much slower for dense features

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Example Task: Predicting Health

- General Philosophy: Extract every feature that might be relevant
- Features for medical diagnosis
    - height
    - weight
    - body temperature
    - blood pressure
    - etc...

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Feature Issues for Linear Predictors

- For linear predictors,it's important **how** features are added
- Three types of nonlinearities can cause problems:
  1. Non-monotonicity
  2. Saturation
  3. Interactions between features

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Non-monotonicity: The Issue

- Feature Map: $\phi(x) = [1, \text{temperature}(x)]$
- Action: Predict health score $y \in \mathbf{R}$ (positive is good)
- Hypothesis Space $\mathcal{F} = \{\text{affine functions of temperature}\}$
- Issue:
    - Health is not an affine function of temperature.
    - Affine function can either say
        - Very high is bad and very low is good, or
        - Very low is bad and very high is good,
        - But here, both extremes are bad.

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Non-monotonicity: Solution 1

- Transform the input:

$$\phi(x) = \left[1, \{\text{temperature(x)-37}\}^2\right],$$

  where 37 is "normal" temperature in Celsius.
- Ok, but requires manually-specified domain knowledge
  - Do we really need that?

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Non-monotonicity: Solution 2

- Think less, put in more:

$$\phi(x) = \left[1, \text{temperature}(x), \{\text{temperature}(x)\}^2\right].$$

- **More expressive** than Solution 1.

General Rule

Features should be simple building blocks that can be pieced together.

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Saturation: The Issue

- Setting: Find products relevant to user's query
- Input: Product x
- Action: Score the relevance of x to user's query
- Feature Map:
$$\phi(x) = [1, N(x)],$$
where $N(x) =$ number of people who bought x.
- We expect a monotonic relationship between $N(x)$ and relevance, but...

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Saturation: The Issue

Is relevance linear in N(x)?

- Relevance score reflects confidence in relevance prediction.
- Are we 10 times more confident if $N(x) = 1000$ vs $N(x) = 100$?

- Bigger is better... but not that much better.

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Saturation: Solve with nonlinear transform

- Smooth nonlinear transformation:

$$\phi(x) = [1, \log\{1 + N(x)\}]$$

- $\log(\cdot)$ good for values with large dynamic ranges
- *Does it matter what base we use in the log?*

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Saturation: Solve by discretization

- Discretization (a discontinuous transformation):

$$\phi(x) = (1(0 \leqslant N(x) < 10), 1(10 \leqslant N(x) < 100), \ldots)$$

- Small buckets allow quite flexible relationship

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Interactions: The Issue

- Input: Patient information x
- Action: Health score $y \in \mathbf{R}$ (higher is better)
- Feature Map

$$\phi(x) = [\text{height}(x), \text{weight}(x)]$$

- Issue: It's the weight **relative** to the height that's important.
- Impossible to get with these features and a linear classifier.
- Need some **interaction** between height and weight.

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Interactions: Approach 1

- Google “ideal weight from height”
- J. D. Robinson’s “ideal weight” formula (for a male):

$$\text{weight(kg)} = 52 + 1.9\,[\text{height(in)} - 60]$$

- Make score square deviation between height(h) and ideal weight(w)

$$f(x) = (52 + 1.9\,[h(x) - 60] - w(x))^2$$

- WolframAlpha for complicated Mathematics:

$$f(x) = 3.61h(x)^2 - 3.8h(x)w(x) - 235.6h(x) + w(x)^2 + 124w(x) + 3844$$

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Interactions: Approach 2

- Just include all second order features:

$$\phi(x) = \left[1, h(x), w(x), h(x)^2, w(x)^2, \underbrace{h(x)w(x)}_{\text{cross term}}\right]$$

- More flexible, no Google, no WolframAlpha.

General Principle

Simpler building blocks replace a single "smart" feature.

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Predicate Features and Interaction Terms

Definition

A **predicate** on the input space $\mathcal{X}$ is a function $P : \mathcal{X} \to \{\text{True}, \text{False}\}$.

- Many features take this form:
  - $x \mapsto s(x) = 1(\text{subject is sleeping})$
  - $x \mapsto d(x) = 1(\text{subject is driving})$
- For predicates, interaction terms correspond to **AND** conjunctions:
  - $x \mapsto s(x)d(x) = 1(\text{subject is sleeping AND subject is driving})$

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So What's Linear?

- Non-linear feature map $\phi : \mathcal{X} \to \mathbf{R}^d$
- Hypothesis space:

$$\mathcal{F} = \left\{ f(x) = w^T \phi(x) \mid w \in \mathbf{R}^d \right\}.$$

- Linear in w? Yes.
- Linear in $\phi(x)$? Yes.
- Linear in x? No.
    - Linearity not even defined unless $\mathcal{X}$ is a vector space

Key Idea: Non-Linearity

- Nonlinear $f(x)$ is important for **expressivity**.
- $f(x)$ linear in w and $\phi(x)$: makes finding $f^*(x)$ much easier

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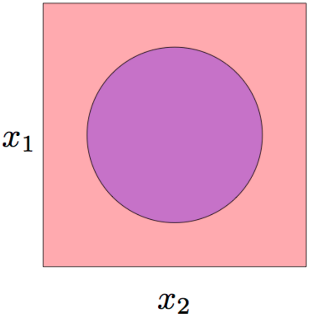
Geometric Example: Two class problem, nonlinear boundary



- With linear feature map $\phi(x) = (x_1, x_2)$ and linear models, no hope
- With appropriate nonlinearity $\phi(x) = \left(x_1, x_2, x_1^2 + x_2^2\right)$, piece of cake.
- Video: `http://youtu.be/3liCbRZPrZA`

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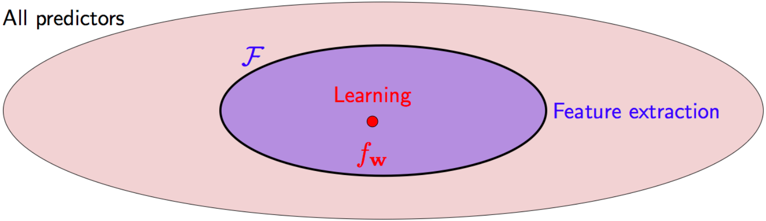
Expressivity of Hypothesis Space

- Consider a linear hypothesis space with a feature map $\phi : \mathcal{X} \to \mathbf{R}^d$:

$$\mathcal{F} = \left\{ f(x) = w^T \phi(x) \right\}$$



Question: does $\mathcal{F}$ contain a good predictor?

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