

Week 4 Lab: Concept Check Exercises

Subgradients

1. (★) If $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is convex and differentiable at x , the $\partial f(x) = \{\nabla f(x)\}$.
2. Fix $f : \mathbb{R}^n \rightarrow \mathbb{R}$ and $x \in \mathbb{R}^n$. Then the subdifferential $\partial f(x)$ is a convex set.
3. (a) True or False: A subgradient of $f : \mathbb{R}^n \rightarrow \mathbb{R}$ at x is normal to a hyperplane that globally underestimates the graph of f .
(b) True or False: If $g \in \partial f(x)$ then $-g$ is a descent direction of f .
(c) True or False: For $f : \mathbb{R} \rightarrow \mathbb{R}$, if $1, -1 \in \partial f(x)$ then x is a global minimizer of f .
(d) True or False: Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ and let $g \in \partial f(x)$. Then $\alpha g \in \partial f(x)$ for all $\alpha \in [0, 1]$.
(e) True or False: If the sublevel sets of a function are convex, then the function is convex.
4. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $f(x_1, x_2) = |x_1| + 2|x_2|$. Compute $\partial f(x_1, x_2)$ for each $x_1, x_2 \in \mathbb{R}^2$.